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THE STABILITY OF GROWING INHOMOGENEOUSLY AGEING VISCOELASTIC BODIES*

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The results in /1, 2/ on the stability of growing viscoelastic rods in finite and infinite time intervals are generalized.

1. Formulation of the problem of the stability of a growing viscoelastic body. We consider a body fabricated at a time $t = 0$ and occupying the domain Ω_0 in three-dimensional space. Continuous growth of the body occurs in the time interval $[t_0, t_1]$, where $t_0 \geqslant 0$. The law of growth, i.e., the dependence of the body configuration on time, is considered to be given. The time of generation of a material particle with coordinates $\mathbf{x} = \{x_i\}$ $(i = 1, 2, 3)$ is denoted by $\tau^* (\mathbf{x})$.

The body is subjected to mass loads F and surface loads q applied to the body boundary $S_q (t), \mathbf{F} = \{F_i\}, \mathbf{q} = \{q_i\}$. Note that the body surface through which growth of the material occurs is part of the surface S_q. On the other part of the body surface $S_n (t)$ we are given the displacements, which to be specific, we set equal to zero. We will later assume that the type of boundary conditions does not change during body fabrication.

Displacements $u_i (t, \mathbf{x})$ governing the unperturbed trajectory motion appears in the body under the action of external forces. We will henceforth assume the growth of the body to occur fairly slowly and the displacements u_i to be slowly varying functions of time, whereupon inertial effects can be neglected.

We assume that during the growth of the body its configuration turns out to be different from the designed one (for instance, the longitudinal axis of a growing rod actually turns out to be curved instead of straight (designed)). This means that the material point coordinates (when there are no external loads) are $x_i + \alpha v_i^\circ$ instead of x_i. We consider v_i° to be fairly small. The parameter α is introduced provisionally, it can be set equal to unity.

In such a body the displacements will equal $u_i^* = u_i + \alpha v_i$.

We will call the body motion governed by the displacements u_i^* perturbed and the displacements αv_i the desired perturbations.

We introduce the displacement norm ($V (t)$ is the body volume at the time t)

$$\| \mathbf{u} (t) \| = \Big(\int\limits_{V(t)} u_i (t, \mathbf{x})\, u_i (t, \mathbf{x})\, dV \Big)^{1/2}$$

Here and henceforth, summation is over repeated subscripts.

Definition. The unperturbed motion of a growing viscoelastic body is called stable in an infinite time interval if for any number $A > 0$ as small as desired there is a number $\delta = \delta (A) > 0$, such that for any initial displacements αv_i° satisfying the inequality $\alpha \| \mathbf{v}^\circ \| < \delta$ and displacements αv_i corresponding to this perturbation will satisfy the inequality $\alpha \| \mathbf{v} \| < A$ for $0 \leqslant t < \infty$.

If the motion of the growing body is investigated in a finite time interval $[0, T]$ and a critical value is given for the displacement norm $\| \mathbf{v} \|^*$, then it is possible to speak of a

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critical time t^* by defining it as the time of first attainment of the displacement norm $\alpha \| \mathbf{v} \|$ by the quantity $\| \mathbf{v} \|^*$: $\alpha \max \| \mathbf{v}(t) \| < \| \mathbf{v} \|^*$, $0 \leqslant t < t^*$, where $\alpha \| \mathbf{v}(t^*) \| = \| \mathbf{v} \|^*$.

We call a body stable in the time interval $[0, T]$ if $t^* > T$.

Assuming the deformations to be small, the equations of state for a material are taken in the form /3/

$$\sigma_{ij} = (E_{ijkl} - \mathbf{R}_{ijkl}) \varepsilon_{kl}, \quad E_{ijkl} = E_{ijkl}(t - \tau^*(\mathbf{x}), \mathbf{x}) \tag{1.1}$$

$$\mathbf{R}_{ijkl}\varepsilon_{kl} = \int\limits_{\tau^*(\mathbf{x})}^{t} \overset{\circ}{R}_{ijkl}\varepsilon_{kl}(\tau, \mathbf{x})\, d\tau$$

$$\overset{\circ}{R}_{ijkl} = R_{ijkl}(t - \tau^*(\mathbf{x}), \tau - \tau^*(\mathbf{x}), \mathbf{x}), \quad t \geqslant \tau^*(\mathbf{x})$$

The relaxation kernels satisfy the conditions

$$0 \leqslant \overset{\circ}{R}_{ijkl} \leqslant R^*_{ijkl}(t, \tau, \mathbf{x}) \tag{1.2}$$

The dependence of $E_{ijkl}, \overset{\circ}{R}_{ijkl}$ on not only $\tau^*(\mathbf{x})$ but also on $\mathbf{x}$ means that the body can be inhomogeneous.

It is easy to see that the deformations at points of the body at a time $t \geqslant \tau^*(\mathbf{x})$ are determined by the expression

$$\varepsilon_{ij}(t, \mathbf{x}) = e_{ij}^{\circ}(\mathbf{x}) + e_{ij}(t, \mathbf{x}) - e_{ij}(\tau^*(\mathbf{x}), \mathbf{x})$$

$$e_{ij}(t, \mathbf{x}) = \frac{1}{2}\{[u^*_{i,j}(t, \mathbf{x}) + u^*_{j,i}(t, \mathbf{x})] - \alpha(\overset{\circ}{v}_{i,j} + \overset{\circ}{v}_{j,i}) + [u^*_{k,i}(t, \mathbf{x})\, u^*_{k,j}(t, \mathbf{x}) - \alpha^2 \overset{\circ}{v}_{k,i} \overset{\circ}{v}_{k,j}]\}$$

$$e_{ij}(\tau^*(\mathbf{x}), \mathbf{x}) = e_{ij}(t, \mathbf{x}) \text{ when } t = \tau^*(\mathbf{x})$$

$e_{ij}^{\circ}(\mathbf{x})$ is the initial deformation of the material particle attached to the body at time $\tau^*(\mathbf{x})$. If the body growth is produced by particles without preliminary stretch then $e_{ij}^{\circ}(\mathbf{x}) \equiv 0$.

Note that the stresses $\sigma_{ij}^{\circ}(\mathbf{x})$ in the attachable particles, due to the strains $e_{ij}^{\circ}(\mathbf{x}) \neq 0$ should be consistent with the boundary conditions on the body surface.

Assuming the external loads are conservative, we write the functional /4/

$$Э = \int\limits_{V(t)} \left[\frac{1}{2} E_{ijkl}\varepsilon_{ij}\varepsilon_{kl} - \varepsilon_{ij}(\mathbf{R}_{ijkl}\varepsilon_{kl})\right] dV - \int\limits_{V(t)} F_i u_i^*\, dV - \int\limits_{S_q(t)} q_i u_i^*\, dS$$

We vary Э with respect to the displacements v_i corresponding to the time t (the displacements u_i corresponding to the unperturbed motion are not varied).

The condition of stationarity of the functional Э is that its first variation equal zero

$$\delta Э = \alpha \delta Э' + \alpha^2 \delta Э'' = 0 \tag{1.3}$$

Here $\delta Э'$, $\delta Э''$ are expressions in the variation $\delta Э$ for appropriate degrees of the parameter α.

Because of the equilibrium of the body in unperturbed motion the equality $\delta Э' = 0$ should hold. Consequently, it follows from (1.3) that

$$\delta Э'' = 0 \tag{1.4}$$

We will henceforth confine ourselves to examining the case when the displacements u_i in the unperturbed motion of a viscoelastic body are small and can be found from the equations of the linear theory of growing media. Then (1.4) can be represented in the form

$$\int\limits_{V(t)} \{\delta v_{i,j}[(E_{ijkl} - \mathbf{R}_{ijkl}) v_{k,l}] + \sigma_{ij}(v_{k,i} + \overset{\circ}{v}_{k,i})\,\delta v_{k,j}\}\, dV = 0 \tag{1.5}$$

where σ_{ij} are stresses in the unperturbed motion and δv_i are variations of the displacements v_i.

We note that (1.5) is a convenient apparatus for determining the characteristics of the state of stress and strain of growing viscoelastic bodies that is a generalization of the Ritz method as it applies to the bodies mentioned.

2. The stability of a body in a finite time interval. We take the displacements v_i as variations of the displacement δv_i. Then (1.5) can be represented in the form

$$(\mathbf{v}', E\mathbf{v}') = -(\mathbf{v}', \sigma\mathbf{v}') - (\mathbf{v}', \sigma\mathbf{v}^{\circ\prime}) + (\mathbf{v}', \mathbf{R}\mathbf{v}') \tag{2.1}$$

$$(\mathbf{v}', E\mathbf{v}') = \int\limits_V E_{ijkl} v_{i,j} v_{k,l}\, dV, \quad (\mathbf{v}', \sigma\mathbf{v}') = \int\limits_V \sigma_{ij} v_{k,i} v_{k,j}\, dV$$

$$(\mathbf{v}',\sigma\mathbf{v}^{\circ\prime})=\int_V \sigma_{ij}\overset{\circ}{v}_{k,i}v_{k,j}\,dV,\quad (\mathbf{v}',\mathbf{R}\mathbf{v}')=\int_V v_{i,j}(\mathbf{R}_{ijkl}v_{k,l})\,dV$$

We assume that the external load is a single-parameter one, whereupon we can write $\sigma_{ij} = -\beta\sigma_{ij}^{\circ}$, $\beta = \text{const}$.

We introduce the vector $\mathbf{v}'$ with the components v_{ij}. We define the scalar product of two vectors $\mathbf{v}_1'$, $\mathbf{v}_2'$ and the norm of the vector $\mathbf{v}'$ as follows

$$(\mathbf{v}_1',\mathbf{v}_2')=\int_V v_{i,j}^{(1)}v_{i,j}^{(2)}\,dV,\quad \|\mathbf{v}'\|=\Big(\int_V v_{i,j}v_{i,j}\,dV\Big)^{1/2}$$

We consider the loads such that the minimum eigenvalue λ_1 of the equation

$$(\mathbf{v}', E\mathbf{v}') = \lambda\,(\mathbf{v}', \sigma^{\circ}\mathbf{v}') \tag{2.2}$$

is positive $(\lambda_1 \geqslant a > 0)$ for any time $t \in [0, T]$.

By expanding the vectors $\mathbf{v}'(t)$ and $\mathbf{v}^{\circ\prime}$ in a complete orthogonal system of functions normalized "with weight σ_{ij}°" corresponding to the eigenvalues λ_i of (2.2), we obtain the following relationship from (2.1):

$$(\lambda_1 - \beta)\,\|\mathbf{v}'\|^2 \leqslant \beta\,\|\mathbf{v}'\|\,\|\mathbf{v}^{\circ\prime}\| + (\mathbf{v}', \mathbf{R}\mathbf{v}') \tag{2.3}$$

Predefining the functions $v_{i,j}(\tau,\mathbf{x})$ for $0 \leqslant \tau < \tau^*(\mathbf{x})$ as $v_{i,j}(\tau,\mathbf{x}) = 0$ and taking condition (1.2) into account, we estimate the right-hand side of the inequality (2.3) as follows:

$$(\mathbf{v}',\mathbf{R}\mathbf{v}') \leqslant \|\mathbf{v}'(t)\|\int_0^t R(t,\tau)\,\|\mathbf{v}'(\tau)\|\,d\tau \tag{2.4}$$

$$R(t,\tau) = \sup_{\mathbf{x}} R_{\max},\; R_{\max} = R_{\max}(t-\tau^*(\mathbf{x}),\, \tau-\tau^*(\mathbf{x}),\, \mathbf{x})$$

where $R_{\max}$ is the maximum eigenvalue of the matrix $\overset{\circ}{R}_{ijkl}$.

Taking (2.4) into account, we have from (2.3)

$$(\lambda_1 - \beta)\,\|\mathbf{v}'(t)\| \leqslant \beta\,\|\mathbf{v}^{\circ\prime}\| + \int_0^t R(t,\tau)\,\|\mathbf{v}'(\tau)\|\,d\tau \tag{2.5}$$

An estimate of the norm $\|\mathbf{v}'(t)\|$ is found from the integral inequality (2.5), as can be done, say, by using the Gronwall-Bellman lemma. On the basis of an imbedding theorem for the norm of the body displacement perturbations $\|\mathbf{v}(t)\|$ the relationships $\|\mathbf{v}(t)\| \leqslant C\,\|\mathbf{v}'(t)\|$ holds, where C is the Korn constant /5/.

An analysis of the stability of a growing body in a finite time interval (/2/, for example) can be performed by using the estimate of the norm $\|\mathbf{v}(t)\|$ obtained in this manner.

3. The stability of a growing body in an infinite time interval. It is assumed further, that starting with the time t_1 the body dimensions and the loads acting on it remain invariant in time while the stresses in unperturbed motion $\sigma_{ij}(t,\mathbf{x})$ tend to limit values equal to $\sigma_{ij}(\mathbf{x})$ as $t \to \infty$. It is also assumed that the elastic moduli E_{ijkl} and the relaxation kernels $\overset{\circ}{R}_{ijkl}$ of the material satisfy the additional conditions

$$\lim_{t\to\infty} E_{ijkl}(t-\tau^*(\mathbf{x}),\mathbf{x}) = \overset{\circ}{E}_{ijkl}(\mathbf{x})$$

$$\int_T^t \sup_{\mathbf{x}} |\overset{\circ}{R}_{ijkl} - R_{ijkl}'(t,\tau,\mathbf{x})|\,d\tau \to 0 \quad \text{as} \quad T\to\infty$$

$$\lim_{T\to\infty}\sup_{t\geqslant T}\int_T^t R_{ijkl}'(t,\tau,\mathbf{x})\,d\tau = \Gamma_{ijkl}'(\mathbf{x})$$

$$\Gamma_{ijkl}^*(\mathbf{x}) = \sup_{t\geqslant 0}\int_0^t R_{ijkl}^*(t,\tau)\,d\tau$$

Theorem. When the formulated conditions as satisfied, a growing body is stable in an infinite time interval if the inequality $\lambda_1^* > \beta$ holds. Here λ_1^* is understood to be the minimum eigenvalue $(\lambda_1^* \geqslant c > 0)$ corresponding to the equation

$$(\mathbf{v}', E^{\circ}\mathbf{v}') = \lambda\,(\mathbf{v}', \sigma^*\mathbf{v}')$$

$$(\mathbf{v}', E^{\circ}\mathbf{v}') = \int_U \overset{\circ}{E}_{ijkl}v_{i,j}v_{k,l}\,dV \quad (\mathbf{v}', \sigma^*\mathbf{v}') = \int_U \sigma_{ij}^* v_{k,i}v_{k,j}\,dV$$

(U is the body volume for $t \geqslant t_1$, $\sigma_{ij}(\mathbf{x}) = -\beta\sigma_{ij}^*(\mathbf{x})$).

The proof of this theorem literally repeats the proof of analogous stability theorems for an inhomogeneously ageing viscoelastic (not growing) body /6/ and a growing rod /2/.

4. Examples. Let us consider two examples that illustrate the importance of taking account of the growth factor for a correct estimate of the serviceability of growing bodies.

Example 1. We consider the problem of the plane state of strain for a body extending infinitely in the direction of the x_3 axis (Fig.1) and growing continuously in horizontal layers at the rate $w(t) = w_0 e^{-\eta t}$. The body material is considered to be isotropic, elastic and ageing. Poisson's ratio is constant with time but the shear modulus is governed by the expression (G_0 and ρ are constants)

$$G = G_0\{1 - \exp[-\rho(t - \tau^*(x_1))]\}, \quad \tau^*(x_1) = -\frac{1}{\eta}\ln\left(1 - \frac{\eta}{w_0}x_1\right)$$

We assume that a uniaxial state of stress

$$\sigma_{11}(t, x_1) = -\gamma[h(t) - x_1] = -p, \quad \gamma = \text{const}$$

is realized in unperturbed motion.

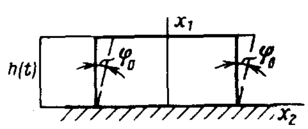

Fig.1

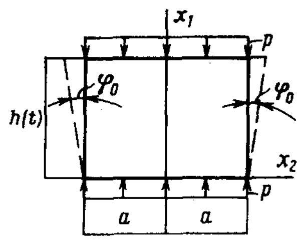

Fig.2

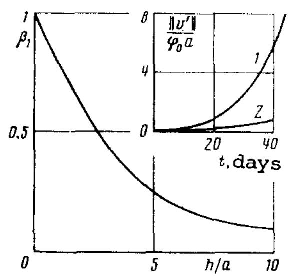

Fig.3

We assume that during growth the side edges of the body received a small deflection from the vertical by a small angle φ_0 ($v_2^\circ = \varphi_0 x_1$). Additional displacements v_1, v_2 appear in the body under the action of a load. The equations

$$\left[\frac{E}{1-\mu^2}(v_{1,1} + \mu v_{2,2})\right]_{,1} + [G(v_{1,2} + v_{2,1})]_{,2} - (pv_{1,1})_{,1} = 0 \tag{4.1}$$

$$[G(v_{1,2} + v_{2,1})]_{,1} + \left[\frac{E}{1-\mu^2}(\mu v_{1,1} + v_{2,2})\right]_{,2} - (pv_{2,1})_{,1} = (p\overset{\circ}{v}_{2,1})_{,1}$$

follow from the variational Eq.(1.5).

The conditions

$$x_1 = 0, \quad v_1 = v_2 = 0; \quad x_1 = h, \quad \sigma_{11} = 0, \quad \sigma_{12} = 0$$

are satisfied on the upper and lower edges of the body.

Here σ_{ij} are stress perturbations.

We assume the body to be stretched in the direction x_2, whereupon the boundary conditions on the side edges can be discarded.

We will seek the solution of (4.1) in the form $v_1 = v_1(x_1), v_2 = v_2(x_1)$. Then

$$\left(\frac{E}{1-\mu^2}v_{1,1}\right)_{,1} - (pv_{1,1})_{,1} = 0; \quad (Gv_{2,1})_{,1} - (pv_{2,1})_{,1} = \varphi_0(p)_{,1}$$

We find the solution of the second of these equations. For $x_1 = h$ the equality $Gv_{2,1} - p(v_{2,1} + \varphi_0) = 0$ should be satisfied, and taking it into account we write

$$v_2 = \varphi_0 \int_0^{x_1} \frac{p}{G - p}\,dx_1$$

The denominator of the integral can vanish for $x_1 = 0$ at a certain time t_+ that is found from the equations

$$G_0[1 - \exp(-\rho t_+)] = \gamma h(t_+), \quad h(t_+) = w_0\eta^{-1}[1 - \exp(-\eta t_+)]$$

The unperturbed motion of the body is obviously stable in an infinite time interval only when for any time $t > 0$

$$1 - e^{-\rho t} > \gamma w_0 G_0^{-1} \eta^{-1} (1 - e^{-\gamma t}) \tag{4.2}$$

Otherwise, it is possible to speak of the stability of the same motion in just a finite time interval. Thus, for a fixed value Δ of the limit deflection of the upper edge of the body in the direction of the x_2 axis, the critical time t_* for the function v_2 to reach the value Δ will be less than the time t_+. As is seen from (4.2), the characteristics of its growth rate play a significant role in estimating the stability of even an elastic body.

Example 2. Let us consider a rectangular body that is infinite in the direction of the x_3 axis and grows continuously in horizontal layers in the direction of the x_1 axis at a constant rate w_0 (Fig.2). The body material is isotropic, viscoelastic, ageing with an elastic modulus that is constant in time, and Poisson's ratio ν. The relaxation kernel for the uniaxial state is taken in the form (γ, b, A, C are constants)

$$\frac{R^\circ}{E} = -\frac{\partial}{\partial \tau} \{\omega(\tau - \tau^*(x_1)) [1 - e^{-\gamma(t-\tau)}]\}$$
$$\omega(\tau - \tau^*(x_1)) = C + A \exp[-b(\tau - \tau^*(x_1))]$$

We will consider that a uniaxial state of stress $\sigma_{11}(t, x_1) = -p$ is realized in the unperturbed motion.

We assume that the side faces of the body have a symmetric deflection by an angle φ_0 ($v_2^\circ = \varphi_0 x_1 x_2 / a$) relative to the x_1 axis. Perturbations of the displacements v_1, v_2 that are determined from (4.1) in which the operators $E(1 - \mathbf{R}), G(1 - \mathbf{R})$ should replace E and G, appear under the action of loads. The boundary conditions

$$x_1 = 0, h, \quad \sigma_{11} = p(v_1 + v_1^\circ)_{,1} \quad v_2 = 0 \tag{4.3}$$
$$x_2 = \pm a, \quad \sigma_{22} = 0, \quad \sigma_{21} = 0 \tag{4.4}$$

are satisfied on the body edges.

Here σ_{ij} are understood to be stress perturbations.

The relationship (2.5) takes the form

$$(1 - \varkappa(t)) \| \mathbf{v}'(t) \| \leqslant \varkappa(t) \| \mathbf{v}^{\circ\prime} \| + \int_0^t R_1(t, \tau) \| \mathbf{v}'(\tau) \| d\tau \tag{4.5}$$
$$R_1(t, \tau) = \frac{R^*(t, \tau)}{E}, \quad \varkappa(t) = \frac{\beta}{\beta_1(t)}, \quad \beta = \frac{p}{G}$$
$$\| \mathbf{v}^{\circ\prime} \| = \varphi_0 \{{}^2/_3 a^{-1} h(t) [h^2(t) + a^2]\}^{1/2}$$

$\beta(t_1)$ is the critical value of the parameter β for an elastic body at a time t.

The solution of the homogeneous system of Eqs.(4.1) for an elastic body under the boundary conditions (4.3) and (4.4) are written as /7/

$$u_1 = (k_1 c_1 \operatorname{sh} k_1 X_2 + k_2 c_2 \operatorname{sh} k_2 X_2) \cos X_1 \tag{4.6}$$
$$u_2 = (k_1^2 c_1 \operatorname{ch} k_1 X_2 + c_2 \operatorname{ch} k_2 X_2) \sin X_1$$
$$k_1^2 = 1 - \frac{1-\mu}{2} \beta, \quad k_2^2 = 1 - \beta, \quad \mu = \frac{\nu}{1-\nu}, \quad X_n = \frac{\pi}{h} x_n, \quad n = 1, 2$$

The value of the parameter β_1 is determined from the characteristic equation that follows from (4.4) after substituting expressions (4.6) therein

$$4 k_1 k_2 \operatorname{th} k_2 \alpha = (2 - \beta)^2 \operatorname{th} k_1 \alpha, \quad \alpha = \pi a / h \tag{4.7}$$

A graph of the change in β_1 with time is shown in Fig.3 for $\mu = 0.5$. Note that

$$h = wat, \quad \alpha = \pi/(wt), \quad w = w_0/a$$

Here the graphs $\| v'(t) \| \sim t$ are represented for a body whose material is characterized by the following constants: C/G=0.075, A/G=0.75, γ=0.02 1/day, b=0.005 1/day for β = 0.1. Curve *1* corresponds to the rate w=0.01 day^{-1} and curve *2* to 0.05 day^{-1}.

The examples considered indicate the substantial influence of the growth rate (fabrication) of the body on its stability and on the magnitude of its displacements.

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THE ORBITAL GEOMETRY OF JUPITER'S MOONS*

M.KH. KHASANOVA

Qualitative geometrical and differential-geometrical characteristics of the orbits of the moons of Jupiter in its gravitational field are studied (up to the fourth zonal harmonic).

Eliminating the cylical coordinate from the energy integral, we determine the domains of possible motions of the moons. The boundaries of these domains are given in the ρz plane by the Hill curves

$$W(\rho, z) + h = 0 \quad (W = U - C^2/(2\rho^2)) \tag{1}$$

Here W is the modified force function, ρ and z are the cylindrical coordinates of the moon, U is the gravitational potential of Jupiter /1/ and C is the area constant. The Hill boundary which is usually considered, derived from the energy integral in its initial form, describes approximately the domain of possible motions in the form of a spheroid, while in the present case the domain in question will be a spheroidal layer.

Let us replace the force function in (1) by its expression /2, 3/

$$U = \frac{fM}{r}\left[1 + \sum_{k=2}^{\infty} I_k \left(\frac{R}{r}\right)^k P_k(\sin\varphi)\right], \quad \sin\varphi = \frac{z}{r} \tag{2}$$

in which f is the gravitational constant, M is the mass of Jupiter, R is the mean equatorial radius, r is the planetocentric distance of the moon, φ is the planetocentric latitude, I_k are dimensionless constants, and $P_k(\sin\varphi)$ is a k-th order Legendre polynomial. When $I_k = 0$, the equation of the Hill curve takes the following form:

$$r_0 = a\,(1 \pm \sqrt{1 - \cos^2 i \sec^2 \varphi}, \quad \cos i = C/(fMa)^{1/2}$$

where a and i denote the major semi-axis and the inclination of the Keplerian orbit. In the case when $I_k \neq 0$, we assign appropriate values to the constants I_k to obtain the domains of possible motions, which can be used to assess the effect of the asphericity of Jupiter on the orbits of the moons.

Retaining in the gravitational potential of Jupiter only the second and fourth zonal harmonics, we shall write the equation of the Hill curve (1) in the form (l is the eccentricity of the orbit)

$$r = r_0 - r_1 I_2 + r_2 I_2^2 + R^4 \frac{5 + \eta\,[8 + 5(7\eta + 5\eta^2)]}{64a^3 S(\xi, \eta)} \times I_4 \tag{3}$$

$$r_1 = \frac{R^2(2 - \eta - 3\eta^2)}{8aS(\xi, \eta)}, \quad r_2 = \frac{R^2 r_1^2 (2 + 2b(1 + \eta)) - 3(1 - e^2)(1 + \xi)}{r_0 S(\xi, \eta)}$$

$$c^2 = {}^1/_2 fMa\,(1 - e^2)(1 + \xi), \quad b = r_0/a, \quad S(\xi, \eta) = {}^1/_2\,[(1 - e^2)(1 + \xi) - b(1 + \eta)]$$

$$\eta = \cos 2\varphi, \quad \xi = \cos 2i$$

The geometrical characteristics of the motion of Jupiter's moons were studied using the values of the astronomical constants given in /1, 3/. Figure 1 shows the domains of possible motions of Jupiter's moons and Fig.2 shows the perturbations in the radius vector of the boundary Hill curve caused by the asphericity of Jupiter (the notation is the same as in Fig. 1). Figure 1 shows that the Hill curves are ovals. Since some of them intersect each other collisions between the moons cannot be ruled out.

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